EN.601.655 – CIS I                                         Department of Computer Science

Fall 2021                                                                                                  Whiting School of Engineering, Johns Hopkins University

***Programming Assignment 1 Report***

***Team Members***

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1. MATHEMATICAL APPROACH

We created four basic customized math packages to help us to accomplish the programming goals, which are called “FrameHomo.m”, “FrameTrans.m”, “InvF.m”, and “TriDxform.m”. Using these four functions, we created two advanced functions, “T2TR.m” and “pivotCalibration.m”, based on specific algorithms to get the required data sets. In this section, instructions on using each function will be introduced and the specific algorithms used in those two advanced functions will be discussed in the next section.

1. **“FrameHomo.m”**

This function is used to get the homogeneous form of a pose when given a rotation matrix and a translation . For example,

Having the homogeneous form of a pose could make calculations of multiple transformations more convenient.

1. **“FrameTrans.m”**

This function accepts two inputs, frame and frame in homogeneous form, and returns three outputs, the rotation matrix and translation vector after transformation and the frame after transformation in homogeneous form. For example,

Where

[1]

1. **“InvF.m”**

This function calculates the inverse of a frame . It accepts one input of frame and returns three outputs, the inversed rotation matrix , the inversed translation vector and the inversed frame . For example,

Where

[1]

1. **“TriDxform.m”**

This function transforms a 3D point into a new coordinate system of frame. It accepts a 3D point and the desired transformation . For example,

Where

[1]

1. **“T2TR.m”**

This function accepts two inputs, the first data set and the second data set , and returns the transformation between the two data sets in homogeneous form by using singular value decomposition (SVD). Herer, we only introduce how to use the T2TR function, and the detailed algorithm will be discussed in the next section. For example,

Where mathematically, .

1. **“pivotCalibration”**

This function accepts one input, frame in homogeneous form, and returns two outputs, an arbitrary translation and the translation of the pivot by using least squares method. As what we talked about at the beginning, the detailed algorithm will be discussed in the next section. Example of using this function is shown below:

***Question 4***

Question 4 is the application of function “T2TR.m”. The expected output for this problem is the pose of calibration object relative to EM tracker base, . Given the data set of the positions of the calibration object LEDs relative to the optical tracker, , and the data set of the positions of LEDs on the calibration object, , the equation of getting the transformation between calibration object and optical tracker coordinate, , is

Similarly, we will have the transformation between optical tracker and EM tracker coordinates, as

Where is the position of LEDs on the EM tracker base, and is the positions of the calibration object LEDs relative to the EM tracker coordinate. Both and are calculated by the function “T2TR.m”. Since we have the two transformations, is just

Where is the measured positions of the EM tracker markers on the calibration object.

***Question 5***

Question 5 is the application of the function “pivotCalibration.m”. Here, we used midpoint method to compute , the unknown but fixed location of pivot of the EM probe. The midpoint of the observed points is calculated as

Where is the number of EM markers on probe and is the position of EM probe relative to EM tracker base. Then we have

And we will use this as the only position vector to compute a transformation by using “T2TR.m” such that

Then we could input into the function “pivotCalibration.m” to get the desired .

***Question 6***

We used a similar method used in Question 5 for Question 6, and the only difference is that in this problem, a coordinate transformation is needed before doing the “pivot” calibration. To transform the coordinate system of the optical probe using “T2TR.m”

Where is the position of optical markers on probe relative to the optical tracker and is the position of optical markers on probe relative to the EM tracker. Then, we could still use the midpoint method to compute the position of unknown but fixed pivot of the optical probe,

The required transformation of optical tracker beacon positions into EM tracker coordinates is computed by using “T2TR.m”.

Then we could input into the function “pivotCalibration.m” to get the desired .

1. ALGORITHMIC STEP

In this assignment, our team develop two algorithms to deal with problem described in the instruction.

**3D to 3D Registration Algorithm**

The input arguments of this algorithm are two point sets noted and . At the beginning, we set the size of each set as N\*3 matrices to make further calculations conveniently. Then, we calculate the midpoints of and

Where N is the number of points in each point set.

Next is to transfer every point of and relative corresponding midpoints.

Furthermore, a direct technique proposed by K.S.Arun [2] is applied to compute rotation matrix .

*Step 1:* Compute

*Step 2:* Compute the SVD (singular value decomposition) of by using the MATLAB function .

*Step 3:* Compute .

*Step 4:* Check if , if it equals to 1, obtain directly. If it equals to -1, update , where is obtained from by inverting the sign of its third column. Otherwise, the algorithm may fail.

*Step 5:* Calculate position vector .

*Step 6:* Output homogeneous representation of transformation frame .

**Pivot Calibration**

*Step 1:* Extract rotation matrix and from each frame.

*Step 2:* Transfer the equation (11) to matrix format

*Step 3:* Compute and by using MATLAB least square function , where .

*Step 4:*  Output and

1. OVERVIEW OF PROGRAM

We put all programming codes for Question 4, 5, and 6 in one file and operators could adjust the variable name at the top of the code to run different data set, as shown in Figure 1.:

图表

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Figure 1. Variables in changing file’s name

Then, the document could generate the output data file, including the results for , , and , no matter the input file is for debugging or for testing (“unknown”). The corresponding code to each question is separated by section notations (“%%”) in MATLAB and operators could follow the “%%%” to see the meaning of each part. The specific equations and mathematical ideas have been illustrated in the **Mathematical Approaches** section. For those debugging files, the following commands shown in Figure 2. could be used to test the accuracy of the program.

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Figure 2. Testing commands

1. STEPS TO VERIFY WORK OF PROGRAM

*Step 1:* Extract expected from our generated output file and example files.

*Step 2:* Since the instructor said that there are some noises in the given data and the distortion can be as much as 2% in the example. Therefore, we compute the difference between each point and compare it with 2% of example data.

*Step 3:* MATLAB will automatically do the comparison and return a logical matrix which only contains 1 and 0. 1 means the statement above is true and 0 means it is incorrect. According to the return matrix of above comparison, we can clearly verify the work of our program.

­*Step 4:* Extract estimated post position with EM probe pivot calibration , which is the data at the second line of generated output file and example file.

*Step 5:* Compute the difference of pivot position between generated output and example file. Then compare the difference to 2% of example data. If it is less than 2% of example data, then our program works correctly, and vice versa.

*Step 6:* Extract estimated post position with optical probe pivot calibration , which is the data at the third line of generated output file and example file.

*Step 7:* Repeat Step 5.

1. TABULAR SUMMARY

Please see the table of unknown output in the APPENDIX section.

1. DISCUSSION OF RESULTS

According to the tabular summary of unknown data and comparison between real output and example data, our team find that in most data sets, results generated by the program are much closed to the example data and the error is within 2% of example which is rational since there is existing noise in the given dataset. In other words, our program has good performance on pivot calibration and 3D to 3D registration. In the further step, we should consider how to remove the noise and generate better result.

1. GROUP CONTRIBUTION

Yuliang Xiao:

1. Discuss with teammate about designing assignment structure.

2. Develop code about algorithm of pivot calibration and 3D to 3D registration.

3. Write the algorithm step, tabular summary, and discussion of results section in the report.

Yuanwu He:

1. Discuss with teammate about designing assignment structure.

2. Debug the program and fix the bugs.

3. Write mathematical approach and overview of computer program structure sections.

1. APPENDIX

**pa1-real-unknown-h-output.txt**

27, 8, pa1-real-unknown-h-output.txt

209.47 195.42 216.93

405.17 406.34 197.55

211.35 209.81 210.77

213.34 206.35 335.71

215.32 202.89 460.65

210.47 334.76 214.25

212.45 331.30 339.18

214.44 327.84 464.12

209.59 459.71 217.72

211.57 456.25 342.66

213.55 452.79 467.59

336.34 210.75 208.82

338.32 207.29 333.75

340.30 203.83 458.69

335.45 335.70 212.29

337.44 332.24 337.23

339.42 328.78 462.16

334.57 460.65 215.76

336.55 457.19 340.70

338.53 453.73 465.64

461.32 211.69 206.86

463.30 208.23 331.80

465.28 204.77 456.73

460.43 336.64 210.33

462.42 333.18 335.27

464.40 329.72 460.21

459.55 461.58 213.81

461.53 458.12 338.74

463.52 454.66 463.68

211.72 211.18 449.35

214.67 212.76 574.31

217.63 214.34 699.26

211.01 336.17 447.79

213.97 337.75 572.74

216.92 339.33 697.70

210.31 461.16 446.22

213.26 462.74 571.18

216.22 464.32 696.13

336.68 211.85 446.39

339.64 213.43 571.35

342.59 215.01 696.30

335.98 336.84 444.83

338.93 338.42 569.78

341.89 340.00 694.74

335.27 461.83 443.26

338.23 463.41 568.22

341.18 464.99 693.17

461.65 212.52 443.43

464.60 214.10 568.38

467.55 215.68 693.34

460.94 337.51 441.86

463.89 339.09 566.82

466.85 340.67 691.77

460.24 462.49 440.30

463.19 464.08 565.25

466.14 465.66 690.21

208.61 451.55 208.70

209.65 451.31 333.70

210.70 451.07 458.70

211.51 576.52 208.92

212.55 576.28 333.91

213.60 576.04 458.91

214.41 701.49 209.13

215.45 701.25 334.13

216.50 701.01 459.13

333.57 448.65 207.65

334.61 448.41 332.65

335.66 448.18 457.65

336.47 573.62 207.87

337.52 573.38 332.87

338.56 573.14 457.86

339.37 698.59 208.09

340.42 698.35 333.08

341.46 698.11 458.08

458.53 445.76 206.61

459.58 445.52 331.60

460.62 445.28 456.60

461.43 570.72 206.82

462.48 570.48 331.82

463.52 570.24 456.81

464.33 695.69 207.04

465.38 695.45 332.03

466.42 695.21 457.03

211.15 449.52 448.73

212.61 450.92 573.72

214.07 452.31 698.70

213.55 574.49 447.31

215.01 575.88 572.30

216.48 577.28 697.28

215.95 699.46 445.89

217.42 700.85 570.87

218.88 702.25 695.86

336.11 447.10 447.30

337.58 448.50 572.28

339.04 449.89 697.27

338.52 572.07 445.88

339.98 573.46 570.86

341.44 574.86 695.84

340.92 697.04 444.45

342.38 698.43 569.44

343.85 699.83 694.42

461.08 444.68 445.86

462.54 446.07 570.85

464.01 447.47 695.83

463.49 569.65 444.44

464.95 571.04 569.42

466.41 572.44 694.41

465.89 694.62 443.02

467.35 696.01 568.00

468.82 697.41 692.98

449.34 210.36 208.84

448.96 211.59 333.83

448.58 212.83 458.82

452.47 335.31 207.61

452.09 336.55 332.61

451.72 337.78 457.60

455.60 460.27 206.39

455.22 461.50 331.38

454.85 462.74 456.37

574.30 207.23 209.24

573.92 208.47 334.24

573.54 209.70 459.23

577.43 332.19 208.02

577.05 333.42 333.01

576.68 334.66 458.01

580.56 457.14 206.79

580.18 458.38 331.79

579.81 459.61 456.78

699.26 204.10 209.65

698.88 205.34 334.64

698.51 206.57 459.64

702.39 329.06 208.43

702.01 330.29 333.42

701.64 331.53 458.41

705.52 454.01 207.20

705.14 455.25 332.19

704.77 456.48 457.19

448.66 209.31 450.31

449.98 210.32 575.30

451.30 211.33 700.29

447.59 334.30 449.32

448.92 335.31 574.31

450.24 336.32 699.30

446.53 459.29 448.32

447.85 460.30 573.31

449.18 461.31 698.30

573.65 210.36 448.98

574.97 211.37 573.97

576.29 212.38 698.96

572.58 335.35 447.99

573.91 336.36 572.98

575.23 337.37 697.96

571.52 460.35 446.99

572.84 461.35 571.98

574.17 462.36 696.97

698.63 211.42 447.65

699.96 212.42 572.64

701.28 213.43 697.63

697.57 336.41 446.66

698.89 337.41 571.64

700.22 338.42 696.63

696.51 461.40 445.66

697.83 462.41 570.65

699.15 463.41 695.64

448.54 449.49 210.79

448.45 450.60 335.78

448.37 451.71 460.78

449.14 574.48 209.68

449.06 575.60 334.67

448.97 576.71 459.67

449.75 699.48 208.57

449.66 700.59 333.56

449.57 701.70 458.56

573.54 448.89 210.88

573.45 450.00 335.88

573.37 451.11 460.87

574.14 573.88 209.77

574.06 574.99 334.77

573.97 576.10 459.76

574.74 698.88 208.66

574.66 699.99 333.65

574.57 701.10 458.65

698.54 448.29 210.97

698.45 449.40 335.97

698.36 450.51 460.96

699.14 573.28 209.86

699.05 574.39 334.86

698.97 575.50 459.85

699.74 698.27 208.75

699.66 699.38 333.75

699.57 700.50 458.74

450.26 451.53 451.04

446.61 451.21 575.98

442.96 450.90 700.93

450.30 576.53 451.35

446.65 576.21 576.30

443.00 575.90 701.24

450.34 701.53 451.66

446.69 701.21 576.61

443.04 700.90 701.56

575.20 451.48 454.68

571.55 451.16 579.63

567.90 450.85 704.58

575.24 576.48 455.00

571.59 576.16 579.95

567.94 575.85 704.89

575.28 701.48 455.31

571.63 701.16 580.26

567.98 700.85 705.21

700.15 451.43 458.33

696.50 451.12 583.28

692.85 450.80 708.23

700.19 576.43 458.65

696.54 576.12 583.59

692.89 575.80 708.54

700.23 701.43 458.96

696.58 701.11 583.91

692.93 700.80 708.86

**pa1-real-unknown-i-output.txt**

27, 8, pa1-real-unknown-i-output.txt

206.30 200.56 194.23

391.88 414.51 202.91

209.49 209.98 209.48

206.93 213.47 334.41

204.37 216.96 459.33

212.79 334.89 206.06

210.23 338.38 330.99

207.66 341.87 455.91

216.08 459.80 202.64

213.52 463.29 327.56

210.96 466.78 452.49

334.42 206.76 212.14

331.86 210.25 337.06

329.30 213.74 461.99

337.72 331.67 208.72

335.16 335.16 333.64

332.59 338.65 458.57

341.01 456.58 205.29

338.45 460.07 330.22

335.89 463.56 455.14

459.35 203.54 214.79

456.79 207.03 339.72

454.23 210.52 464.64

462.65 328.45 211.37

460.09 331.94 336.29

457.52 335.43 461.22

465.94 453.36 207.95

463.38 456.85 332.87

460.82 460.34 457.80

208.54 211.46 449.48

211.43 210.10 574.44

214.33 208.74 699.40

207.14 336.45 450.87

210.04 335.09 575.83

212.93 333.73 700.79

205.74 461.43 452.27

208.64 460.07 577.22

211.53 458.71 702.18

333.50 212.89 446.60

336.39 211.53 571.56

339.29 210.17 696.52

332.10 337.88 447.99

334.99 336.52 572.95

337.89 335.15 697.91

330.70 462.86 449.39

333.60 461.50 574.34

336.49 460.14 699.30

458.45 214.32 443.72

461.35 212.96 568.68

464.24 211.60 693.64

457.06 339.31 445.11

459.95 337.94 570.07

462.85 336.58 695.03

455.66 464.29 446.51

458.56 462.93 571.46

461.45 461.57 696.42

208.60 451.81 211.71

205.22 451.71 336.66

201.83 451.61 461.61

207.06 576.80 211.76

203.67 576.70 336.72

200.29 576.60 461.67

205.52 701.79 211.82

202.13 701.69 336.77

198.74 701.59 461.73

333.55 453.35 215.09

330.16 453.25 340.05

326.77 453.15 465.00

332.01 578.34 215.15

328.62 578.24 340.10

325.23 578.14 465.06

330.46 703.33 215.21

327.08 703.23 340.16

323.69 703.13 465.11

458.49 454.89 218.48

455.11 454.79 343.44

451.72 454.69 468.39

456.95 579.88 218.54

453.56 579.78 343.49

450.18 579.68 468.45

455.41 704.87 218.59

452.02 704.77 343.55

448.63 704.67 468.50

210.37 448.51 449.10

209.58 449.56 574.10

208.78 450.61 699.09

207.16 573.46 448.04

206.36 574.51 573.03

205.57 575.56 698.02

203.95 698.42 446.97

203.15 699.47 571.96

202.36 700.51 696.95

335.33 451.73 449.87

334.53 452.78 574.87

333.74 453.83 699.86

332.12 576.68 448.80

331.32 577.73 573.80

330.52 578.78 698.79

328.90 701.64 447.74

328.11 702.69 572.73

327.31 703.73 697.72

460.28 454.95 450.64

459.49 456.00 575.63

458.69 457.04 700.63

457.07 579.90 449.57

456.28 580.95 574.57

455.48 582.00 699.56

453.86 704.86 448.50

453.06 705.90 573.50

452.27 706.95 698.49

450.76 210.41 211.09

448.31 210.08 336.06

445.86 209.75 461.04

450.35 335.41 211.41

447.90 335.08 336.38

445.45 334.75 461.36

449.93 460.40 211.73

447.48 460.07 336.71

445.03 459.74 461.68

575.74 210.81 213.54

573.29 210.48 338.51

570.83 210.15 463.49

575.32 335.81 213.86

572.87 335.48 338.83

570.42 335.15 463.81

574.91 460.81 214.18

572.46 460.48 339.16

570.01 460.15 464.13

700.71 211.22 215.99

698.26 210.89 340.96

695.81 210.56 465.94

700.30 336.22 216.31

697.85 335.89 341.29

695.40 335.56 466.26

699.88 461.22 216.63

697.43 460.89 341.61

694.98 460.56 466.58

450.29 211.43 451.61

447.87 208.03 576.55

445.45 204.63 701.48

448.40 336.37 454.98

445.98 332.97 579.91

443.56 329.57 704.84

446.51 461.31 458.35

444.09 457.91 583.28

441.67 454.51 708.21

575.25 213.26 454.08

572.83 209.86 579.01

570.42 206.45 703.95

573.36 338.20 457.45

570.94 334.80 582.38

568.52 331.39 707.31

571.47 463.14 460.82

569.05 459.74 585.75

566.63 456.33 710.68

700.22 215.09 456.55

697.80 211.68 581.48

695.38 208.28 706.42

698.32 340.03 459.92

695.90 336.62 584.85

693.48 333.22 709.78

696.43 464.97 463.29

694.01 461.57 588.22

691.59 458.16 713.15

448.53 449.30 209.40

448.65 450.23 334.39

448.78 451.16 459.39

446.09 574.27 208.47

446.21 575.20 333.46

446.34 576.13 458.46

443.65 699.25 207.54

443.77 700.18 332.54

443.90 701.11 457.53

573.51 451.74 209.25

573.63 452.67 334.25

573.76 453.60 459.25

571.06 576.71 208.32

571.19 577.64 333.32

571.31 578.58 458.32

568.62 701.69 207.40

568.75 702.62 332.39

568.87 703.55 457.39

698.48 454.18 209.11

698.61 455.11 334.11

698.73 456.04 459.10

696.04 579.15 208.18

696.17 580.08 333.18

696.29 581.02 458.17

693.60 704.13 207.25

693.72 705.06 332.25

693.85 705.99 457.25

448.61 449.16 450.61

451.17 445.87 575.54

453.73 442.58 700.47

451.06 574.09 453.85

453.62 570.80 578.78

456.18 567.51 703.71

453.51 699.02 457.09

456.07 695.73 582.02

458.63 692.44 706.96

573.56 446.77 447.99

576.12 443.48 572.92

578.68 440.19 697.85

576.01 571.71 451.23

578.57 568.42 576.16

581.13 565.12 701.09

578.46 696.64 454.47

581.02 693.35 579.40

583.58 690.06 704.33

698.51 444.39 445.37

701.07 441.10 570.30

703.63 437.81 695.23

700.96 569.32 448.61

703.52 566.03 573.54

706.08 562.74 698.47

703.41 694.26 451.85

705.97 690.96 576.78

708.53 687.67 701.71

**pa1-real-unknown-j-output.txt**

27, 8,pa1-real-unknown-j-output.txt

191.07 190.53 210.24

397.66 408.18 202.79

209.46 210.62 209.24

210.77 213.25 334.20

212.07 215.87 459.17

208.36 335.59 206.62

209.66 338.22 331.59

210.97 340.84 456.56

207.26 460.56 204.01

208.56 463.18 328.98

209.87 465.81 453.94

334.45 211.70 207.91

335.76 214.32 332.87

337.06 216.95 457.84

333.35 336.67 205.30

334.65 339.29 330.26

335.96 341.92 455.23

332.24 461.64 202.68

333.55 464.26 327.65

334.85 466.88 452.62

459.44 212.78 206.58

460.74 215.40 331.55

462.05 218.02 456.51

458.34 337.74 203.97

459.64 340.37 328.93

460.95 342.99 453.90

457.23 462.71 201.36

458.54 465.33 326.32

459.84 467.96 451.29

210.54 211.04 449.79

213.29 212.75 574.75

216.04 214.47 699.70

209.95 336.02 448.09

212.70 337.74 573.04

215.45 339.46 698.00

209.36 461.01 446.38

212.11 462.73 571.34

214.86 464.44 696.30

335.51 211.59 447.03

338.26 213.31 571.99

341.01 215.02 696.95

334.92 336.58 445.33

337.67 338.29 570.29

340.42 340.01 695.25

334.33 461.57 443.63

337.08 463.28 568.58

339.83 465.00 693.54

460.48 212.14 444.28

463.23 213.86 569.23

465.98 215.58 694.19

459.89 337.13 442.57

462.64 338.85 567.53

465.39 340.56 692.49

459.30 462.12 440.87

462.05 463.83 565.83

464.79 465.55 690.79

209.81 451.65 210.21

206.55 449.77 335.15

203.29 447.89 460.09

212.00 576.61 212.14

208.74 574.73 337.09

205.48 572.85 462.03

214.19 701.58 214.08

210.93 699.70 339.03

207.67 697.82 463.97

334.75 449.41 213.43

331.49 447.53 338.38

328.23 445.65 463.32

336.94 574.38 215.37

333.68 572.50 340.31

330.42 570.61 465.26

339.13 699.34 217.31

335.87 697.46 342.25

332.61 695.58 467.20

459.69 447.17 216.66

456.43 445.29 341.60

453.17 443.41 466.55

461.88 572.14 218.60

458.62 570.26 343.54

455.36 568.38 468.49

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213.92 574.92 450.47

215.49 574.15 575.46

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218.89 699.10 576.19

220.47 698.32 701.18

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575.17 212.22 212.14

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578.33 453.27 704.39

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**pa1-real-unknown-k-output.txt**

27, 8,pa1-real-unknown-k-output.txt

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335.57 460.72 461.34

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460.57 460.68 461.23

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695.87 703.36 451.84

692.89 704.80 576.80

689.90 706.25 701.76

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